Void Fraction Prediction in Two-Phase Flow Across a Tube Bundle

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The hydrodynamics of two-phase flow has been investigated extensively for in-tube flows and for parallel flows in tube bundles. Very limited work has been performed, however, on vertical two-phase flows across tube bundles, which occur frequently on the shell side of heat exchangers such as kettle reboilers used in the process and other industries. In recent years, a few articles have been published on the measurement and prediction of void fraction in adiabatic, vertical air-water flow across horizontal rod bundles (Kondo and Nakajima, 1980; Schrage et al., 1988; Dowlati et al., 1990).

A reliable void fraction model is essential to the accurate evaluation of the gravitational component of total pressure drop and circulation flow rates in kettle reboilers. Dowlati et al. (1990) proposed an empirical correlation for void fraction as a function of the dimensionless gas velocity, j_g^*, based on air/water experiments performed with in-line bundles. That correlation was found to account for the mass velocity effects and to apply to different rod bundles with varying pitch, rod diameter, and layout (Dowlati et al., 1992).

This work examines void fraction prediction using the phenomenological approach developed by Zuber and Findlay (1965), which is commonly referred to as the drift flux model. This approach is also compared to the void fraction prediction using an empirical correlation given earlier by Dowlati et al. (1990).

Drift Flux Model

Zuber and Findlay (1965) derived the following general expression for the weighted mean velocity, based on the drift flux model,

$$\overline{u}_g = C_o\langle j\rangle + \overline{V}_{gj} \tag{1}$$

where $\langle j\rangle$ is the mixture mean velocity based on the total volumetric flow rate and the minimum flow area, and $\overline{V}_{gj}$ is called the drift velocity.

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This expression takes into account the effect of nonuniform velocity and void profiles by using the distribution parameter, C_o, and the effect of local relative velocity between the phases due to slip by the drift velocity.

Although the above expression was derived for general two-phase flow, this approach, to our knowledge, has never been applied to void fraction prediction for two-phase cross flow in tube bundles. Following the definitions of Zuber and Findlay (1965), the weighted mean velocity of the gas phase can be defined as,

$$\overline{u}_g = \frac{\langle j_g\rangle}{\langle\alpha\rangle} \tag{2}$$

Substituting Eq. 2 into Eq. 1, we obtain an expression for the average void fraction,

$$\langle\alpha\rangle = \frac{\langle j_g\rangle}{C_o\langle j\rangle + \overline{V}_{gj}} \tag{3}$$

In analyzing several data sets from two-phase flow in tubes, Zuber and Findlay plotted $\overline{u}_g$ against $\langle j\rangle$ and observed that the data followed closely the linear relationship given by Eq. 1. Thus, the values of C_o and $\overline{V}_{gj}$ could be taken as the slope and y-intercept, respectively.

In this work, Eq. 1 was tested against the data sets collected from adiabatic, vertical two-phase flow of the air-water system across horizontal rod bundles (Dowlati et al., 1990, 1992) at near atmospheric pressures. The rod bundle geometries and test conditions used for comparison are listed in Table 1. All bundles were rectangular in cross-section and consisted of 5 columns × 20 rows of 80-mm-long acrylic rods. Half-rods were used at the channel walls to minimize by-pass flow. Air was injected into the water stream through a porous tube and the resulting two-phase flow mixture flowed through a flow straightner upstream of the bundle. Void fraction was locally measured at different elevations using a gamma densitometer placed on a traversable platform. Then, the local void fraction values were averaged to yield reliable bundle average void fraction data.

The data for all six test bundles are shown in Figure 1. The

Table 1. Rod Bundle Geometries and Test Conditions (Dowlati et al., 1990, 1992)

Bundle Geometry	Rod Dia. (mm)	P/D	G (kg/m^2·s)	Flow Quality
In-line*	19.05	1.3	27–818	0–0.33
In-line*	12.7	1.75	90–542	0–0.08
Staggered**	19.05	1.3	92–795	0–0.15
Staggered**	12.7	1.75	56–538	0–0.13
In-Line†	12.7	1.75 Transverse 2.17 Longitudinal	70–542	0–0.15
In-Line‡	12.7	1.75 Transverse 1.33 Longitudinal	58–542	0–0.2

* Square
** Equilateral triangle
† Rectangle (long)
‡ Rectangle (short)

average gas velocity, $\bar{u}_g$, is based on the minimum flow area and bundle-average void fraction, α. Using linear regression, a best-fit curve was obtained as follows,

$$\bar{u}_g = 1.1035 <j> + 0.33 \qquad (4)$$

Here, both the average gas velocity and mixture mean velocity are specified in m/s. The inset in Figure 1 shows more closely the data scatter in the lower region.

A number of interesting observations can be made regarding Figure 1. First, a single linear fit covers a wide range of $\bar{u}_g$ and $<j>$ values. The range of $\bar{u}_g$ and $<j>$ values covered here are larger than any of the tube flow data tested by Zuber and Findlay (1965), thus showing a wide applicability of the drift flux approach.

Also, the value of C_o obtained from the linear fit is very close to the reported values for flows in circular tubes, which ranged from 1.0 to 1.3 (Nicklin et al., 1962; Zuber and Findlay, 1965; Lahey, 1974; Ishii, 1977; Clark et al., 1990). The C_o value obtained from Figure 1 implies that the void fraction profile is fairly flat across the bundle. One would expect a fairly flat void profile for cross flow in bundles, since the rods enhance turbulent mixing, and this will tend to disperse the gas phase.

Zuber and Findlay also suggested that the drift flux model can be used to identify bubbly flow from slug flow by observing variation in the slope of the plotted data. From Figure 1, there is no apparent slope change in the plotted data set, suggesting the existence of only one flow pattern. Study of flow behavior recorded using a video camera revealed that nearly all test conditions resulted in dispersed bubbly or churn-turbulent bubbly flow. It also is physically difficult for slug flow to exist in such closely spaced rod bundles due to the potential breakup of the gas slugs upon impact on the rods.

The value of $\bar{V}_{gj}$ taken from Figure 1 is also similar to that obtained by Smissaert (1963) for adiabatic air-water flow in a vertical 2-in. (51-mm) ID pipe. It was reported by Zuber and Findlay to be in the range 0.25–0.3 m/s.

Although Zuber and Findlay have proposed equations for determining $\bar{V}_{gj}$ for different flow patterns in a tube, these equations were not found to be applicable to the data set in Figure 1. One reason may be that traditionally $\bar{V}_{gj}$ has been considered to be a function of the terminal rise velocity of a bubble in a stagnant liquid column. Obviously, this velocity would have a different value for a bubble rising through a bank of tubes, especially one with a staggered array, instead of a stagnant liquid. More data using other fluids are necessary before a general expression can be proposed for determination of $\bar{V}_{gj}$ in a tube bundle.

Equation 3 was used to calculate the void fraction in the drift flux model. The values of C_o and $\bar{V}_{gj}$ deduced from Figure 1 were used in Eq. 3. Figure 2 shows a comparison of the predicted void fractions with the experimental data. The average deviation was found to be 11.1%.

It is noted that the drift flux model can account for mass velocity and test bundle geometry effects over the range of conditions covered. Figure 3 shows the void fraction-quality data reported by Dowlati et al. (1990) for one of the test bundles showing the mass velocity effect. Using C_o and $\bar{V}_{gj}$ values obtained from Figure 1, the void fractions as predicted by the drift flux model are also plotted in the same figure (solid line). Clearly the drift flux model is capable of accounting for the mass velocity effect observed in the data.

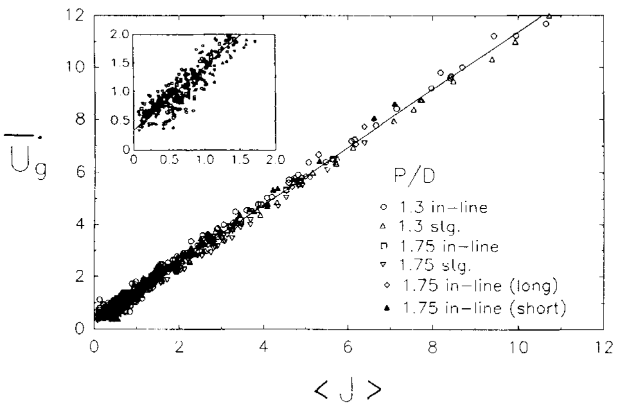

Figure 1. Correlation of void fraction data using drift flux model.

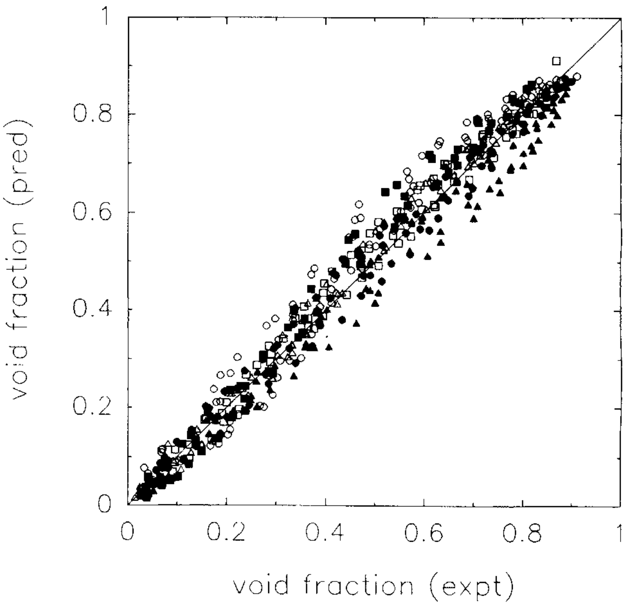

Figure 2. Comparison of predicted and measured void fractions.

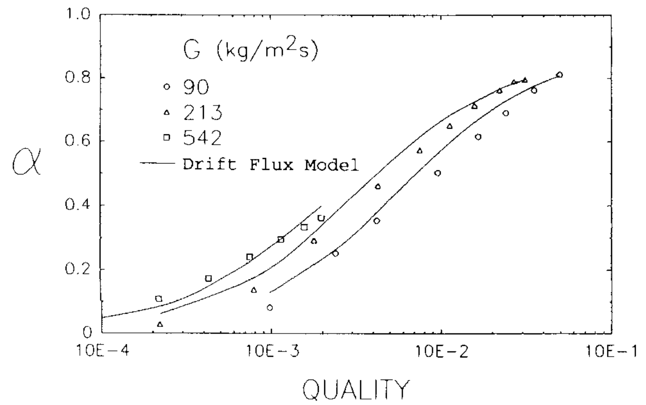

Figure 3. Prediction of mass velocity effects on void fraction.

Comparison with Previous Correlation

Next, the drift flux model is compared with the empirical correlation developed by Dowlati et al. (1990) for two-phase flow across a tube bundle,

$$\alpha = 1 - \frac{1}{\sqrt{1 + C_1 j_g^* + C_2 j_g^{*2}}} \tag{5}$$

where the dimensionless gas velocity, j_g^*, is defined by Wallis (1969) as:

$$j_g^* = \frac{\rho_g^{1/2} < j_g >}{\sqrt{gD(\rho_l - \rho_g)}} \tag{6}$$

where the gas superficial velocity, $<j_g>$, is defined in terms of the minimum flow area, the rod diameter is used for D, and the gas density is evaluated at the average test section pressure. The constants, C_1 and C_2, are chosen to be 35 and 50, respectively, to best fit the overall data set (Dowlati et al., 1992). This correlation can predict the present data set with an average deviation of 10.3%, which is slightly smaller than that obtained with the drift flux model.

To compare the two void fraction correlations more closely their functional behavior was compared, since both equations involve similar parameters. To do this, Eq. 3 was written as a function of mass velocity and quality by substituting the following expressions,

$$<j_g> = \frac{Gx}{\rho_g} \tag{7}$$

$$<j> = G\left[\frac{x}{\rho_g} + \frac{1-x}{\rho_l}\right], \tag{8}$$

into Eq. 3 to obtain:

$$<\alpha> = \frac{1}{C_o\left[1 + \frac{(1-x)}{x}\frac{\rho_g}{\rho_l}\right] + \frac{\overline{V}_{gj}\rho_g}{Gx}} \tag{9}$$

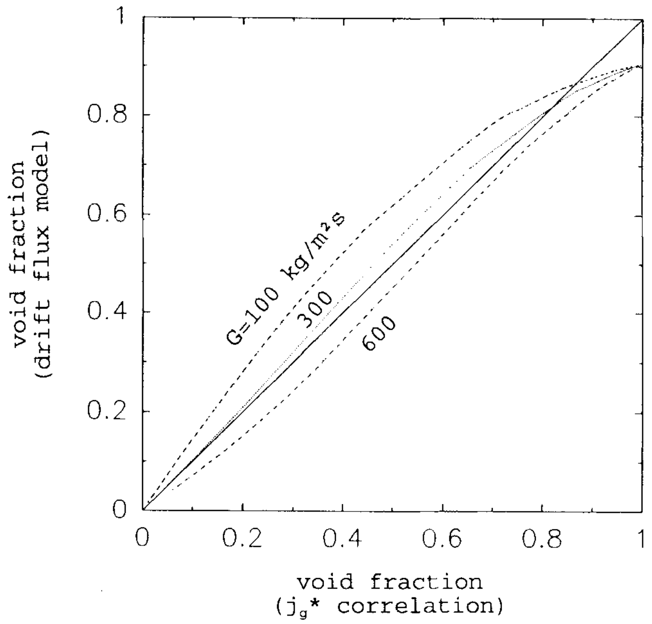

Figure 4. Comparison of drift flux model and Eq. 5 ($C_1 = 35$, $C_2 = 50$) for air/water system.

Similarly, Eq. 5 can be expressed in terms of mass velocity and flow quality by expressing j_g^* in an alternate form,

$$j_g^* = \frac{Gx}{\sqrt{\rho_g g D(\rho_l - \rho_g)}} \tag{10}$$

In Figures 4 and 5, Eqs. 5 and 9 are plotted for air-water and R-113 liquid-vapor systems, respectively, at atmospheric pressure over the range of mass velocities covered experimen-

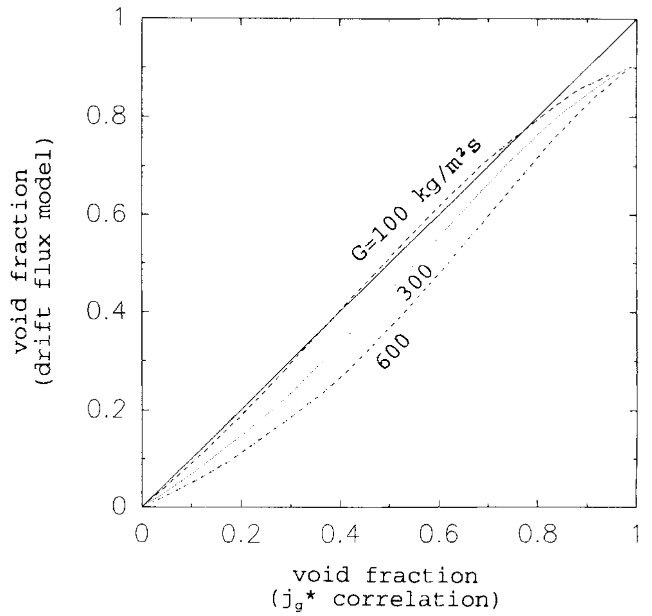

Figure 5. Comparison of drift flux model and Eq. 5 ($C_1 = 35$, $C_2 = 50$) for R-113 system.

tally by Dowlati et al. (1990, 1992). Figure 4 shows that the functional dependences of the two void fraction correlations on mass flux and quality are similar for the case of air/water system at 1 atm. For R-113, Figure 5 shows that the difference between the two correlations is again small, but not the same as that in air-water flow. This slight difference between the predictions for the air/water system and R-113 may be attributed to the differences in fluid densities as well as the values of $\overline{V}_{gj}$, C_1 and C_2, which have been chosen to fit the air/water data.

Finally, it is noted that the drift flux model (Eq. 3) and the empirical correlation, Eq. 5, given by Dowlati et al. (1990), both incorporate two constants that still need to be determined empirically for specific fluid and operating conditions. Future development of a correlation for the drift velocity, $\overline{V}_{gj}$, based on the bubble rise velocity in a tube bundle, requires further experiments involving different fluids and bundle geometries.

Conclusions

The drift flux model was applied to predict the bundle-average void fraction in two-phase flow across a tube bundle. A satisfactory agreement was found with the experimental data for both in-line and staggered bundles using empirically determined values of the distribution parameter, C_o, and drift velocity, $\overline{V}_{gj}$, which are similar to those reported for round tubes.

The drift flux model was also compared with an empirical correlation proposed by Dowlati et al. (1990). Both void fraction equations were shown to be functionally similar and to predict the experimental void fraction data with similar average deviations.

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Notation

C_o = distribution parameter
C_1, C_2 = coefficients, Eq. 5
D = tube or rod diameter, m
g = acceleration of gravity, m/s^2
G = mass velocity based on minimum flow area, kg/m^2·s
$\langle j \rangle$ = mixture mean velocity, m/s
$\langle j_g \rangle$ = gas superficial velocity, m/s
j_g^* = dimensionless gas velocity
P = pitch, m
$\overline{u}_g$ = $Gx/\rho_g\alpha$, gas velocity, m/s
$\overline{V}_{gj}$ = drift velocity, m/s
x = quality
$\langle \ \rangle$ = average over flow cross-section

Greek letters

α = void fraction
ρ_g = gas density, kg/m^3
ρ_l = liquid density, kg/m^3

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